

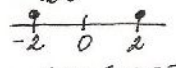
5. Rovnice v součinném tvaru

Příklady

1) Řeš v R

a) $(x-2)(x+3)=0 \Leftrightarrow \overset{a=0}{x-2=0} \vee \overset{b=0}{x+3=0}$
 $x=2 \in \mathbb{R} \quad x=-3 \in \mathbb{R}$
 $\mathcal{K} = \{2, -3\}$

b) $x^2 - 4 = 0 \quad \sigma = \mathbb{R}, \mathcal{D} = \mathbb{R}$
 ~~$(x-2)(x+2)=0$~~
 $x-2=0 \quad x+2=0$
 $x_1=2 \in \mathbb{R} \quad x_2=-2 \in \mathbb{R} \quad \mathcal{K} = \{2, -2\}$

2. upr. $x^2 = 4 \quad | \vee$
 $|x| = 2$

 $\mathcal{K} = \{-2, 2\}$
! POZOR - ČASTO CHYBNĚ !
 ~~$x^2 = 4 \quad | \vee$~~
 ~~$x = 2$~~
ZTRATILI JSME KOŘEN $x = -2$
 $\sqrt{x^2} = |x|$ proto

c) $3y^2 - 6y = 4y^2 - 3y$
 $0 = 4y^2 + 3y$
 $4y^2 + 3y = 0$
 ~~$y(4y+3)=0$~~
 $y=0 \in \mathbb{R} \quad 4y+3=0$
 $4y=-3$
 $y_2 = -\frac{3}{4} \in \mathbb{R} \quad \mathcal{K} = \{0, -\frac{3}{4}\}$

! POZOR - ČASTO CHYBNĚ !
 ~~$4y^2 + 3y = 0 \quad | : y$~~
 ~~$4y + 3 = 0$~~
 ~~$4y = -3$~~
 ~~$y = -\frac{3}{4}$~~
ZTRATILI JSME KOŘEN $y = 0$

ROVNICE V SOUČINOVÉM TVARU - kde, ať už jakýmkoli úpravami převeš na tvar součinu několika výrazů rovnajícího se NULE

- vyhov. $a \cdot b = 0 \Leftrightarrow a=0 \vee b=0$

součin několika čísel (výrazů) je roven 0 $\Leftrightarrow$ alespoň jeden (ty jeden nebo více) je čísel (číslo, výraz) je roven 0

- ROVNICI UPRAVÍME NA ANULOVANÝ TVAR A PAK NA SOUČIN
 - vyhov. rozložíme, vyčísleme, rozdělíme, rozdělíme,...

- ŘEŠENÍM JE SJEDNOTENÍ možných kořenů dílčích rovnic $\mathcal{K} = \mathcal{K}_1 \cup \mathcal{K}_2$

Příklady

2) Řeš v N

a) v N: $9 - d^2 = d + 3 \quad \sigma = \mathbb{N}, \mathcal{D} = \mathbb{N}$
 $0 = d^2 + d - 6$
 $d^2 + d - 6 = 0$
(kde rozdělíme - dělení možností)
[hl. dvě čísla dle, aby součet byl 1, součin -6]
 $(d+3)(d-2)=0$
 $d+3=0 \quad d-2=0$
 $d_1 = -3 \notin \mathbb{N} \quad d_2 = 2 \in \mathbb{N} \quad \mathcal{K} = \{2\}$
POZOR - V JAKÉ MN. ŘEŠÍME

2. upr. $9 - d^2 = d + 3 \quad \sigma = \mathbb{N}, \mathcal{D} = \mathbb{N}$
 $(3-d)(3+d) = d+3$
 $(3-d)(3+d) - d - 3 = 0$
 $(3-d)(3+d) - (d+3) = 0$
 ~~$(3+d)[(3-d)-1] = 0$~~
 ~~$(3+d)(2-d) = 0$~~
 $3+d=0 \quad 2-d=0$
 $d_1 = -3 \notin \mathbb{N} \quad d_2 = 2 \in \mathbb{N}$

$$b) \text{vZ: } (10x^2 - \frac{9}{4})(1-x^2) = 0$$

$$\sigma = \mathbb{Z}, \mathbb{D} = \mathbb{Z}$$

$$(4x - \frac{9}{2})(4x + \frac{9}{2})(1-x)(1+x) = 0$$

$$\begin{array}{l} 4x - \frac{9}{2} = 0 \quad 4x + \frac{9}{2} = 0 \quad 1-x = 0 \quad 1+x = 0 \\ 4x = \frac{9}{2} \quad 4x = -\frac{9}{2} \quad -x = -1 \quad x = -1 \in \mathbb{D} \\ x = \frac{9}{8} \in \mathbb{D} \quad x = -\frac{9}{8} \in \mathbb{D} \quad x = 1 \in \mathbb{D} \quad x = -1 \in \mathbb{D} \end{array}$$

$$\left[\begin{array}{l} \text{poznor!} \\ 4x = \frac{9}{2} \quad 1:4 \quad \text{nebo} \quad 4x = \frac{9}{2} \quad 1:2 \\ \left[\begin{array}{l} x = \frac{9}{8} : 4 \\ x = \frac{9}{2} : \frac{1}{4} \\ x = \frac{9}{8} \end{array} \right] \quad \begin{array}{l} 8x = 3 \quad 1:8 \\ x = \frac{3}{8} \end{array} \end{array} \right]$$

$$\mathcal{K} = \{1, -1\}$$

$$c) \text{vR: } (x^2 - 2)(2\sqrt{3} - 6y) = 0$$

$$\sigma = \mathbb{R}, \mathbb{D} = \mathbb{R}$$

$$(x - \sqrt{2})(x + \sqrt{2})(2\sqrt{3} - 6y) = 0$$

$$\begin{array}{l} x - \sqrt{2} = 0 \quad x + \sqrt{2} = 0 \quad 2\sqrt{3} - 6y = 0 \\ x = \sqrt{2} \in \mathbb{D} \quad x = -\sqrt{2} \in \mathbb{D} \quad 2\sqrt{3} = 6y \quad 1:6 \\ y = \frac{2\sqrt{3}}{6} \\ y = \frac{\sqrt{3}}{3} \in \mathbb{D} \end{array}$$

$$\mathcal{K} = \{\sqrt{2}, -\sqrt{2}, \frac{\sqrt{3}}{3}\}$$

③ Řeš v R

$$a) 4x^3 - x = 0 \quad \sigma = \mathbb{R}, \mathbb{D} = \mathbb{R}$$

$$x(4x^2 - 1) = 0$$

$$x(2x-1)(2x+1) = 0$$

$$\begin{array}{l} x = 0 \in \mathbb{D} \quad 2x-1 = 0 \quad 2x+1 = 0 \\ 2x = 1 \quad 2x = -1 \\ x = \frac{1}{2} \in \mathbb{D} \quad x = -\frac{1}{2} \in \mathbb{D} \end{array}$$

$$\mathcal{K} = \{0, \frac{1}{2}, -\frac{1}{2}\}$$

$$b) 3x^3 = \frac{1}{3}x \quad \sigma = \mathbb{R}, \mathbb{D} = \mathbb{R}$$

$$9x^3 = 1$$

$$9x^3 - 1 = 0$$

$$x(9x^2 - 1) = 0$$

$$\begin{array}{l} x = 0 \in \mathbb{D} \quad 9x^2 - 1 = 0 \\ 9x^2 = 1 \\ x = \frac{1}{3} \in \mathbb{D} \end{array}$$

$$\mathcal{K} = \{0, \frac{1}{3}\}$$

$$c) x^3 - 4x + x^2 - 4 = 0 \quad \sigma = \mathbb{R}, \mathbb{D} = \mathbb{R}$$

$$x(x^2 - 4) + x^2 - 4 = 0$$

$$(x^2 - 4)(x+1) = 0$$

$$(x-2)(x+2)(x+1) = 0$$

$$\begin{array}{l} x-2 = 0 \quad x+2 = 0 \quad x+1 = 0 \\ x = 2 \in \mathbb{D} \quad x = -2 \in \mathbb{D} \quad x = -1 \in \mathbb{D} \end{array}$$

$$\mathcal{K} = \{2, -2, -1\}$$

$$\left[\begin{array}{l} \text{nebo} \\ x^3 - 4x + x^2 - 4 = 0 \\ x^2(x+1) - 4x - 4 = 0 \\ x^2(x+1) - 4(x+1) = 0 \\ (x+1)(x^2 - 4) = 0 \text{ ald.} \end{array} \right]$$

$$d) x^3 + 2x^2 - x - 2 = 0 \quad \sigma = \mathbb{R}, \mathbb{D} = \mathbb{R}$$

$$x^2(x+2) - 1(x+2) = 0$$

$$(x+2)(x^2 - 1) = 0$$

$$(x+2)(x-1)(x+1) = 0$$

$$\begin{array}{l} x+2 = 0 \quad x-1 = 0 \quad x+1 = 0 \\ x = -2 \in \mathbb{D} \quad x = 1 \in \mathbb{D} \quad x = -1 \in \mathbb{D} \end{array}$$

$$\mathcal{K} = \{-2, 1, -1\}$$

$$\left[\begin{array}{l} \text{nebo} \\ x^3 + 2x^2 - x - 2 = 0 \\ x(x^2 - 1) + 2(x^2 - 1) = 0 \\ (x^2 - 1)(x+2) = 0 \text{ ald.} \end{array} \right]$$

4) řešit

a) $\mathbb{N} \setminus \{0\}$: $(x+2)x = (2x-1)(x+2)$

$$(x+2)x - (2x-1)(x+2) = 0$$

$$(x+2)[x - (2x-1)] = 0$$

$$(x+2)(-x+1) = 0$$

$$\begin{array}{l} x+2=0 \\ x=-2 \notin \mathbb{N} \end{array} \quad \begin{array}{l} -x+1=0 \\ x=1 \in \mathbb{N} \end{array}$$

$$\mathcal{K} = \{1\}$$

$$0 = \mathbb{N}, \mathbb{D} = \mathbb{N}$$

[vidíme, že by $(x+2)$ šlo po anulování vykrátit, musíme dělit $(x+2)$]

2. zp. $(x+2)x = (2x-1)(x+2)$

$$x^2 + 2x = 2x^2 - 4x - x - 2$$

$$x^2 + 2x = 2x^2 + 3x - 2 \quad | -x^2 - 2x$$

$$0 = x^2 + x - 2 \quad [\text{kv. v. Malom m. m.}]$$

konstanta krad. rozdělení
(součet 1, součin -2)

$$0 = (x+2)(x-1)$$

$$\begin{array}{l} x+2=0 \\ x=-2 \notin \mathbb{N} \end{array} \quad \begin{array}{l} x-1=0 \\ x=1 \in \mathbb{N} \end{array}$$

b) $\mathbb{N} \setminus \{0\}$: $3x^3 + 5x^2 - 2x = 2x^3 - 8x$

$$0 = \mathbb{N}$$

$$\mathbb{D} = \mathbb{N}$$

$$x^3 + 5x^2 + 6x = 0$$

$$x(x^2 + 5x + 6) = 0$$

rozklad kv. troj.:

$$x(x+3)(x+2) = 0$$

$$\begin{array}{l} x=0 \\ 0 \in \mathbb{N} \end{array} \quad \begin{array}{l} x+3=0 \\ x=-3 \notin \mathbb{N} \end{array} \quad \begin{array}{l} x+2=0 \\ x=-2 \notin \mathbb{N} \end{array}$$

$$\mathcal{K} = \emptyset$$

c) $\mathbb{N} \setminus \{0\}$: $(x^2 - x)(2x^2 - 1) = 0$

$$0 = \mathbb{Z}, \mathbb{D} = \mathbb{Z}$$

$$x(x^2 - 1)(\sqrt{2}x - 1)(\sqrt{2}x + 1) = 0$$

$$x(x-1)(x+1)(x\sqrt{2}-1)(x\sqrt{2}+1) = 0$$

$$\begin{array}{l} x=0 \\ 0 \in \mathbb{N} \end{array} \quad \begin{array}{l} x-1=0 \\ x=1 \in \mathbb{N} \end{array} \quad \begin{array}{l} x+1=0 \\ x=-1 \in \mathbb{Z} \end{array} \quad \begin{array}{l} x\sqrt{2}-1=0 \\ x\sqrt{2}=1 \\ x=\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \\ x=\frac{\sqrt{2}}{2} \notin \mathbb{N} \end{array} \quad \begin{array}{l} x\sqrt{2}+1=0 \\ x\sqrt{2}=-1 \\ x=-\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \\ x=-\frac{\sqrt{2}}{2} \in \mathbb{Z} \end{array}$$

$$\mathcal{K} = \{0, 1, -1\}$$

ne řešit s podmínkou

$$(x^2 - x)(2x^2 - 1) = 0$$

$$x^3 - x = 0$$

$$x(x^2 - 1) = 0$$

$$x(x-1)(x+1) = 0$$

$$\begin{array}{l} x=0 \\ 0 \in \mathbb{N} \end{array} \quad \begin{array}{l} x-1=0 \\ x=1 \in \mathbb{N} \end{array} \quad \begin{array}{l} x+1=0 \\ x=-1 \in \mathbb{Z} \end{array}$$

$$x(x^2 - 1) = 0$$

$$\begin{array}{l} x=0 \\ 0 \in \mathbb{N} \end{array} \quad \begin{array}{l} x^2 - 1 = 0 \\ x^2 = 1 \\ |x| = 1 \end{array} \rightarrow \begin{array}{l} \frac{1}{-1} \quad \frac{1}{0} \quad \frac{1}{1} \\ x_2 = -1 \quad x_3 = 1 \end{array}$$

$$2x^2 - 1 = 0$$

$$(x\sqrt{2}-1)(x\sqrt{2}+1) = 0$$

$$x\sqrt{2} = 1$$

$$x = \frac{1}{\sqrt{2}}$$

$$x_4 = \frac{\sqrt{2}}{2}$$

$$x\sqrt{2} = -1$$

$$x = -\frac{1}{\sqrt{2}}$$

$$x_5 = -\frac{\sqrt{2}}{2}$$

$$\text{nebo } 2x^2 = 1$$

$$x^2 = \frac{1}{2}$$

$$|x| = \sqrt{\frac{1}{2}}$$

$$|x| = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$|x| = \frac{\sqrt{2}}{2}$$

$$-\frac{\sqrt{2}}{2} \quad 0 \quad \frac{\sqrt{2}}{2}$$

$$x_4 = -\frac{\sqrt{2}}{2} \quad x_5 = \frac{\sqrt{2}}{2}$$

5) Řešiv R

a) $x(x+2)=0$ $\sigma=R$
 $\emptyset=R$
 $x=0 \in \emptyset$ $x+2=0$
 $x_2=-2 \in \emptyset$
 $\mathcal{K}=\{0, -2\}$

b) $(2x+1)(\frac{1}{2}x-4)=0$ $\sigma=R$
 $\emptyset=R$
 $2x+1=0$ $\frac{1}{2}x-4=0 \quad | \cdot 2$
 $2x=-1$ $x-8=0$
 $x_1=-\frac{1}{2} \in \emptyset$ $x_2=8 \in \emptyset$
 $\mathcal{K}=\{-\frac{1}{2}, 8\}$

c) $(4-2y)(-3y-4)=0$ $\sigma=R$
 $\emptyset=R$
 $4-2y=0$ $-3y-4=0$
 $4=2y \quad | :2$ $-4=3y \quad | :3$
 $y_1=2 \in \emptyset$ $y_2=-\frac{4}{3} \in \emptyset$
 $\mathcal{K}=\{2, -\frac{4}{3}\}$

d) $(2\sqrt{3}-6y)(3y+\frac{2}{3})=0$
 $2\sqrt{3}-6y=0$ $3y+\frac{2}{3}=0 \quad | \cdot 3$
 $2\sqrt{3}=6y \quad | :6$ $12y+2=0$
 $\frac{2\sqrt{3}}{6}=y$ $12y=-2$
 $y_1=\frac{\sqrt{3}}{3} \in \emptyset$ $y_2=-\frac{1}{6} \in \emptyset$
 $\mathcal{K}=\{\frac{\sqrt{3}}{3}, -\frac{1}{6}\}$

6) Řešiv R

a) $3x^2-1=x-1$ $\sigma=R$
 $\emptyset=R$
 $3x^2-x=0$
 $x(3x-1)=0$
 $x=0 \in \emptyset$ $3x-1=0$
 $3x=1$
 $x_2=\frac{1}{3} \in \emptyset$
 $\mathcal{K}=\{0, \frac{1}{3}\}$

b) $2x^2-4x=6x$ $\sigma=R$
 $\emptyset=R$
 $2x^2-10x=0 \quad | :2$
 $x^2-5x=0$
 $x(x-5)=0$
 $x=0 \in \emptyset$ $x-5=0$
 $x_1=0 \in \emptyset$ $x_2=5 \in \emptyset$
 $\mathcal{K}=\{0, 5\}$

c) $y\sqrt{2}=y^2$ $\sigma=R$
 $\emptyset=R$
 $0=y^2-y\sqrt{2}$
 $y^2-y\sqrt{2}=0$
 $y(y-\sqrt{2})=0$
 $y=0 \in \emptyset$ $y-\sqrt{2}=0$
 $y_1=0 \in \emptyset$ $y_2=\sqrt{2} \in \emptyset$
 $\mathcal{K}=\{0, \sqrt{2}\}$

d) $28=7y^2$ $\sigma=R$
 $\emptyset=R$
 $4=y^2$
 $|y|=2$
 $y=2$ $y=-2$
 $y_1=2 \in \emptyset$ $y_2=-2 \in \emptyset$
 $\mathcal{K}=\{2, -2\}$

e) $2x^2=\frac{1}{2}$ $\sigma=R$
 $\emptyset=R$
 $4x^2=1$
 $|x|=\sqrt{\frac{1}{4}}$
 $|x|=\frac{1}{2}$
 $x=\frac{1}{2}$ $x=-\frac{1}{2}$
 $x_1=\frac{1}{2} \in \emptyset$ $x_2=-\frac{1}{2} \in \emptyset$
 $\mathcal{K}=\{\frac{1}{2}, -\frac{1}{2}\}$

$4x^2-1=0$
 $(2x-1)(2x+1)=0$
 $2x-1=0$ $2x+1=0$
 $2x=1$ $2x=-1$
 $x=\frac{1}{2}$ $x=-\frac{1}{2}$
 $x_1=\frac{1}{2} \in \emptyset$ $x_2=-\frac{1}{2} \in \emptyset$
 $\mathcal{K}=\{\frac{1}{2}, -\frac{1}{2}\}$

f) $3x^2+2=-6x^2+4$ $\sigma=R$
 $\emptyset=R$
 $9x^2-2=0$
 $(3x-\sqrt{2})(3x+\sqrt{2})=0$
 $3x-\sqrt{2}=0$ $3x+\sqrt{2}=0$
 $3x=\sqrt{2}$ $3x=-\sqrt{2}$
 $x=\frac{\sqrt{2}}{3} \in \emptyset$ $x=-\frac{\sqrt{2}}{3} \in \emptyset$

2.kp. $9x^2=2$
 $x^2=\frac{2}{9}$
 $|x|=\sqrt{\frac{2}{9}}$
 $|x|=\frac{\sqrt{2}}{3}$
 $x=\frac{\sqrt{2}}{3}$ $x=-\frac{\sqrt{2}}{3}$
 $x_1=\frac{\sqrt{2}}{3} \in \emptyset$ $x_2=-\frac{\sqrt{2}}{3} \in \emptyset$

PŘI ŘEŠENÍ KONKRETNÍ ROVNICE LZE UŽÍT RŮZNÉ ZPŮSOBY - NÁMĚŠ POUKAZ NA KONKRETNÍ PŘÍKLADY

DALŠÍ PŘÍKLADY NEREŠENÉ - VIZ NÁMĚŠ LIST S PŘÍKLADY